Neighboring Optimal Solution for Fuzzy Travelling Salesman Problem

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Abstract - A new method is introduced to find fuzzy optimal solution for fuzzy Travelling salesman problems. In this method, intuitionistic trapezoidal fuzzy numbers are utilized to find the fuzzy optimal solution. This proposed method provides some of other fuzzy salesman problem very neighbour optimal solution called fuzzy "neighbouring optimal" salesman. A relevant numerical example is also included

Key words - Intuitionistic fuzzy number, intuitionistic trapezoidal fuzzy number, fuzzy salesman algorithm, fuzzy optimal solution

1. INTRODUCTION

Travelling salesman problem is a well-known NP-hard problem in combinatorial optimization. In the ordinary form of travelling salesman problem, a map of cities is given to the salesman and he has to visit all the cities only once and return to the starting point to complete the tour in such a way that the length of the tour is the shortest among all possible tours for this map. The data consists of weights assigned to the edges of a finite complete graph and the objective is to find a cycle passing through all the vertices of the graph while having the minimum total weight. There are different approaches for solving travelling salesman problem. Almostevery new approach for solving engineering and optimization problems has been tried on travelling salesman problem. Many methods have been developed for solving travelling salesman problem. These methods consist of heuristic methods and population based optimization algorithms etc. Heuristic methods like cutting planes and branch and bound can optimally solve only small problems whereas the heuristic methods such as 2-opt, 3-opt, Markov chain, simulated annealing and tabu search are good for large problems. Population based optimization algorithms are a kind of nature based optimization algorithms. The natural systems and creatures which are working and developing in nature are one of the interesting and valuable sources of inspiration for designing and inventing new systems and algorithms in different fields of science and technology. Particle Swarm Optimization, Neural Networks, Evolutionary Computation, Ant Systems etc. are a few of the problem solving techniques inspired from observing nature. Travelling salesman problems in crisp and fuzzy environment have received great attention in recent years [1-4, 5,6,7,8,9,10,11]. With the use of LR fuzzy numbers, the computational efforts required to solve fuzzy assignment problems and fuzzy travelling salesman problem are considerably reduced [12].

In this paper, we introduce new method for finding a fuzzy optimal solution as well as of alternative solutions which is very near to fuzzy optimal solution for the given fuzzy travelling salesman problem. In section 2 ,recall the definition of intuitionistic trapezoidal fuzzy number and some operations. In section 3 , we presented fuzzy travelling salesman problem and algorithm. In section 4 –numerical example. In section 5 , conclusion is also included.

2. PRELIMINARIES

In this section, some basic definitions and arithmetic operations are reviewed.

2.1. INTUITIONISTIC FUZZY NUMBER

Let a set X be fixed an Ifs $A^{\sim i}$ in X is an object having the form $A^{\sim i} = \{(X, \mu_{A^{\sim i}}\ (x), \vartheta_{A^{\sim i}}\,(x))/x \in X\}$ where $\mu_{A^{\sim i}}(x): X \to [0,1]$ and $\vartheta_{A^{\sim i}}(x): X \to [0,1]$,define the degree of membership and degree of non-membership respectively of the element x∈X to the set $A^{\sim i}$,which is a subset of X, for every element of x∈X , $0 \leq \mu_{A^{\sim i}}(x) + \vartheta_{A^{\sim i}}(x) \leq 1$.

2.2. DEFINITION

A IFS $A^{\sim i}$, defined on the universal set of real numbers R, is said to be generalized IFN if its membership and non-membership function has the following characteristics:

(i) $\mu_{A^{\sim i}}(x) : R \to [0, 1]$ is continuous.

(ii) $\mu_{A^{\sim i}}(x) = 0$ for all $x \in (-\infty, a_1] \cup [a_4, \infty)$.

(iii) $\mu_{A^{\sim i}}(x)$ is strictly increasing on $[a_1,a_2]$ and strictly decreasing on $[a_3,a_4]$.

(iv) $\mu_{A^{\sim i}}(x) = w_1$ for all $x \in [a_2,a_3]$.

(v) $\nu_{A^{\sim i}}(x) : R \to [0, 1]$ is continuous.

(vi) $\nu_{A^{\sim i}}(x) = w_2$ for all $x \in [b_2,b_3]$.

(vii) $\nu_{A^{\sim i}}(x)$ is strictly decreasing on $[b_1,b_2]$ and strictly increasing on $[b_3,b_4]$.

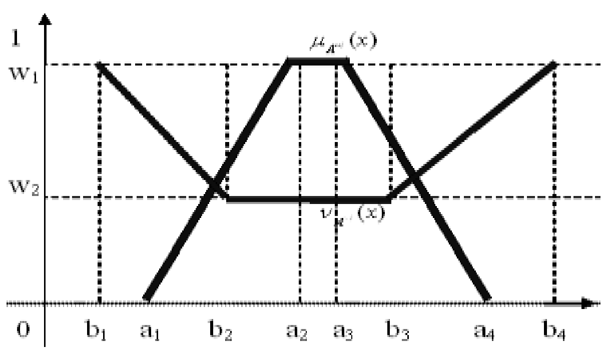
(viii) $\nu_{A^{\sim i}}(x) = w_1$, for all $x \in (-\infty, b_1] \cup [b_4, \infty)$ and $w = w_1+w_2$, $0 < w \leq 1$.

2.3. DEFINITION

A generalized intuitionistic fuzzy number $A^{\sim i}$ is said to be a generalized trapezoidal intuitionistic fuzzy number with parameters, $b_1 \leq a_1 \leq b_2 \leq a_2 \leq a_3 \leq b_3 \leq a_4 \leq b_4$ and denoted by $A^{\sim i} = (b_1,a_1,b_2,a_2,a_3,b_3,a_4,b_4;w_1,w_2)$ if its membership and non-membership function is given by

$$\mu_{A^{\sim i}}(x) = \begin{cases} \frac{w_1(x-a_1)}{(a_2-a_1)}, & a_1 \leq x \leq a_2 \\ w_1, & a_2 \leq x \leq a_3 \\ \frac{w_1(x-a_4)}{(a_3-a_4)}, & a_3 \leq x \leq a_4 \\ 0, & otherwise \end{cases} \quad \text{and} \quad \nu_{A^{\sim i}}(x) = \begin{cases} \frac{w_2(b_2-x)}{(b_2-b_1)}, & b_1 \leq x \leq b_2 \\ w_2, & b_2 \leq x \leq b_3 \\ \frac{w_2(x-b_3)}{(b_4-b_3)}, & b_3 \leq x \leq b_4 \\ w_1, & otherwise \end{cases}$$

Generalized trapezoidal intuitionistic fuzzy number is denoted by $A^{\sim i}_{GITrFN} = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4; w_1, w_2)$. Fig.1 membership and non-membership function of GITrFN.



2.4 DEFINITION

We define a ranking function$\Re$: F(R) → R which maps each fuzzy number in to the real line; F(R) represents the set of all intuitionistic trapezoidal fuzzy numbers. If $\Re$ be any linear ranking function, then $\Re(A^{\sim i}) = \frac{b_1+a_1+b_2+a_2+a_3+b_3+a_4+b_4}{8}$..

2.5 ARITHMETIC OPERATIONS

In this section, arithmetic operations between two intuitionistic trapezoidal fuzzy numbers, defined on universal set of real numbers R. Let $A^{\sim i} = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)$ and $B^{\sim i} = (d_1, c_1, d_2, c_2, c_3, d_3, c_4, d_4)$ intuitionistic trapezoidal fuzzy numbers, are as follows:

- Image$A^{\sim i} = (-b_4, -a_4, -b_3, -a_3, -a_2, -b_2, -a_1, -b_1)$.
- $A^{\sim i} + B^{\sim i} = (b_1 + d_1, a_1 + c_1, b_2 + d_2, a_2 + c_2, a_3 + c_3, b_3 + d_3, a_4 + c_4, b_4 + d_4$.
- $A^{\sim i} - B^{\sim i} = (b_1 - d_4, a_1 - c_4, b_2 - d_3, a_2 - c_3, a_3 - c_2, b_3 - d_2, a_4 - c_1, b_4 - d_1)$.
- if λ is any scalar , then $\lambda A^{\sim i} = (\lambda b_1, \lambda a_1, \lambda b_2, \lambda a_2, \lambda a_3, \lambda b_3, \lambda a_4, \lambda b_4). if\ \lambda > 0.$

  $= (\lambda b_4, \lambda a_4, \lambda b_3, \lambda a_3, \lambda a_2, \lambda b_2, \lambda a_1, \lambda b_1). if\ \lambda < 0.$
- $A^{\sim i} \times B^{\sim i} = (b_1\sigma, a_1\sigma, b_2\sigma, a_2\sigma, a_3\sigma, b_3\sigma, a_4\sigma, b_4\sigma)\ if\ \Re(B^{\sim i}) > 0$

  $= (\ b_4\sigma, a_4\sigma, b_3\sigma, a_3\sigma, a_2\sigma, b_2\sigma, a_1\sigma, b_1\sigma), if\ \Re(B^{\sim i}) < 0$
- $A^{\sim i} \div B^{\sim i} = \left(\frac{b_1}{\sigma}\frac{a_1}{\sigma}, \frac{b_2}{\sigma}, \frac{a_2}{\sigma}, \frac{a_3}{\sigma}, \frac{b_3}{\sigma}, \frac{a_4}{\sigma}, \frac{b_4}{\sigma}\right) if \Re(B^{\sim i}) \neq 0\ \Re(B^{\sim i}) > 0,$

  $= (\frac{b_4}{\sigma}, \frac{a_4}{\sigma}, \frac{b_3}{\sigma}, \frac{a_3}{\sigma}, \frac{a_2}{\sigma}, \frac{b_2}{\sigma}, \frac{a_1}{\sigma}, \frac{b_1}{\sigma})\ if \Re(B^{\sim i}) \neq 0, \Re(B^{\sim i}) < 0.$

  Where $\sigma = (d_1 + c_1 + d_2 + c_2 + c_3 + d_3 + c_4 + d_4)/8$.

3. FUZZY TRAVELLING SALESMAN PROBLEMS

The fuzzy travelling sales man problem is very similar to the fuzzy assignment problem expect that in the former, there is an additional restrictions. Suppose a fuzzy salesman has to visit n cities. He wishes to start from a particular city, visit each city once, and then return to his starting point.The objective is to select the sequence in which the cities are visited in such a way that his total fuzzy travelling time is minimized. Since the salesman has to visit all n cities , the fuzzy optimal solution remains independent of selection of starting point.

The mathematical form of the fuzzy travelling salesman is given below

Minimize $z^{\sim i} = \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} d_{ij}^{\sim i} x_{ijk}^{\sim i}$ i≠j

Subject to

$\sum_{i=1}^{n}\sum_{j=1}^{n} x_{ijk}^{\sim i} = 1$, i≠j, k=1,2,…m

$\sum_{j=1}^{n}\sum_{k=1}^{n} x_{ijk}^{\sim i} = 1$, i=1,2,…m

$\sum_{i=1}^{n}\sum_{k=1}^{n} x_{ijk}^{\sim i} = 1, \qquad j=1,2,\ldots n$

$\sum_{i\neq j} x_{ijk}^{\sim i} = \sum_{i\neq j} x_{ij(k+1)}^{\sim i}$ for all j and k

$$x_{ijk}^{\sim i} = \begin{cases} 1 \;, if\ kth\ directed\ arc\ is\ from\ city\ i\ to\ j \\ 0, \qquad\qquad\qquad\qquad\qquad other\ wise \end{cases}$$

Where i,j and k are integers that vary between 1 and n.

An fuzzy assignment in a row is said to be a minimum fuzzy assignment if the fuzzy cost of the fuzzy assignment is minimum in the row.

A tour of a fuzzy travelling salesman problem is said to be minimum tour if it contains one or more minimum fuzzy assignments.

3.1 ALGORITHM

Step 1 Find the minimum assignments for each row in the fuzzy cost matrix below and above of leading diagonal elements.

Step 2 Find all possible minimum tour and their fuzzy costs.

Step 3: Find the minimum of the all fuzzy costs of the minimum possible tours say $z^{\sim i}$.

Step 4: The tour corresponding to $z^{\sim i}$ is the fuzzy optimal tour and $z^{\sim i}$ is the fuzzy optimal value of the tour.

4. EXAMPLE

Consider the following fuzzy travelling salesman problem so as to minimize the fuzzy cost cycle.

	A	B	C	D
A	-	(-3,-1,0,2,3,4,5,6)	(1,2,3,4,6,7,8,9)	(-10,-6,5,6,10,15,17,19)
B	(-3,-1,0,2,3,4,5,6)	-	(-3,0,2,3,4,5,6,7)	(-6,4,6,8,10,12,14,16)
C	(1,2,3,4,6,7,8,9)	(-3,0,2,3,4,5,6,7)	-	(0,1,2,3,5,6,7,8)
D	(-10,-6,5,6,10,15,17,19)	(-6,4,6,8,10,12,14,16)	(0,1,2,3,5,6,7,8)	-

The minimum fuzzy costs in each row and their elements are given below

R($c_{12}^{\sim i}$) = 2 R($c_{13}^{\sim i}$) = 5 R($c_{14}^{\sim i}$) = 7 R($c_{21}^{\sim i}$) = 2 R($c_{23}^{\sim i}$) = 3 R($c_{24}^{\sim i}$) = 8 R($c_{31}^{\sim i}$) = 5 R($c_{32}^{\sim i}$) = 3 R($c_{34}^{\sim i}$) = 4. R($c_{41}^{\sim i}$) = 7 R($c_{42}^{\sim i}$) = 8 R($c_{43}^{\sim i}$) = 4

1st row $c_{12}^{\sim i}$: AB   2nd row $c_{23}^{\sim i}$: BC   3rd row $c_{34}^{\sim i}$: CD

All possible cycles which contains one or more minimum elements are given below

Cycle	1	2	3	4
1	AB	BC	CD	DA
2	AB	BD	DC	CA
3	AC	CB	BD	DA
4	AC	CD	DB	BA
5	AD	DC	CB	BA
6	AD	DB	BC	CA

The fuzzy cost of the each of the minimum tours with their minimum elements is given below

Cycle	Tour	$z^{\sim i}$	R($z^{\sim i}$)
1	A→B→ $c \to D \to A$	(-16,-6,9,14,22,30,35,40)	16
2	A→B→ $D \to C \to A$	(-8,6,11,17,24,29,34,39)	19
3	A→C→ $B \to D \to A$	(-18,0,16,21,30,39,45,51)	23
4	A→C→ $D \to B \to A$	(-8,6,11,17,24,29,34,39)	19
5	A→D→ $c \to B \to A$	(-16,-6,9,14,22,30,35,40)	16
6	A→D→ $B \to C \to A$	(-18,0,16,21,30,39,45,51)	23

Best tours are A→B→ $c \to D \to A$ and A→D→ $c \to B \to A$.The minimum total distance travelled is 16.

Satisfaction tours are A→B→ $D \to C \to A$ and A→C→ $D \to B \to A$.The total distance travelled is 19.

The worst tours are A→C→ $B \to D \to A$ and A→D→ $B \to C \to A$ The total distance travelled is 23.

5. CONCLUSION

Using the proposed method , we can solve a fuzzy travelling salesman problem. The proposed method is very easy to understand and apply and also provides not only to an fuzzy optimal solution for the problem and also, to list some other alternative solutions to the problem which are very near to fuzzy optimal solution of the problem.

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