Face Recognition using Laplace Beltrami Operator by Optimal Linear Approximations

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Abstract—We propose an appearance-based face recognition technique called the laplacian face method. With Locality Preserving Projections (LPP), the face imageries are mapped into a face subspace for analysis. Unlike from Principal Component Analysis (PCA) and Linear Discriminant Analysis (LDA) which commendably see merely the Euclidean structure of face space, LPP discovers a set in subspace that conserves native information, and finds a face subspace that best perceives the essential face manifold structure. The laplacian faces are the optimal linear approximations to the Eigen functions of the Laplace beltrami operator on the face manifold. In this way, the undesirable variations resulting from changes in lighting, facial expression, and pose may be removed or reduced. Theoretical analysis shows that PCA, LDA, and LPP can be obtained from different graph models. We equate the proposed Laplacian face approach with Eigen face and Fisher face methods on three different face data sets. Experimental results recommend that the proposed Laplacian face method provides a better representation and attains lower error rates in face recognition.

Keywords—Face recognition, principal component analysis, linear discriminant analysis, locality preserving projections, face manifold, subspace learning.

1 Introduction

Lots of face recognition methods have been developed over the former few years. One of the best popular and well-studied practices for face recognition is the appearance-based method [28], [16]. By means of appearance-based methods, we generally characterize an image of size n × m pixels by a vector in an n × m-dimensional space. In fact, these n×m-dimensional spaces are too huge to permit robust and fast face recognition. A common manner to attempt to determine this problem is to use dimensionality reduction methods [7], [9], [6], [10], [16], [15], [21],[28], [29], [37], [37]. Two of the most popular methods for this purpose are Principal Component Analysis (PCA) [26] and Linear Discriminant Analysis (LDA) [3]. PCA is an Eigenvector technique intended to model linear difference in high-dimensional data. PCA achieves dimensionality reduction by projecting the unique n-dimensional data onto the k (<< n)-dimensional linear subspace covered by the foremost Eigenvectors of the data's covariance matrix. Its aim is to discover a set of communally orthogonal basis functions that capture the directions of maximum variance in the data and for which the coefficients are pairwise decorrelated. For linearly fixed manifolds, PCA is sure to find the dimensionality of the manifold and produces a compacted representation. Turk and Pentland[29] use Principal Component Analysis to define face images in terms of a set of basic functions, or "Eigen faces."

LDA is a managed learning algorithm. LDA examines for the project axes on which the data points of various classes are far away from each other while needing data points of the same class to be convenient to each other. Different from PCA which encodes data in an orthogonal linear space, LDA encodes discriminating information in a linearly independent space using bases that are not essentially orthogonal. It is mostly believed that algorithms based on LDA are superior to those based on PCA. However, some modern work [14] demonstrates that, when the training data set is lesser, PCA can outperform LDA, and also that PCA is less delicate to different training data sets. In recent times, a lots of research efforts have shown that the face images possibly exists on a nonlinear sub manifold [7], [10], [18], [19], [21], [23], [27]. However, both PCA and LDA excellently see only the Euclidean structure. They fail to find the underlying structure, if the face images lie on a nonlinear sub manifold hidden in the image space.

In this paper, we propose a new method to face analysis (representation and recognition), which openly considers the manifold structure. To be particular, the manifold structure is modelled by a nearest-neighbour graph which preserves the local structure of the image space. A face subspace is obtained by Locality Preserving Projections (LPP) [9]. Each face image in the image space is plotted to a low-dimensional face subspace, which is considered by a set of feature images, called Laplacian faces. The face subspace preserves native structure and seems to have more discriminating power than the PCA approach for sorting purpose. We also offer theoretical analysis to show that PCA, LDA, and LPP can be obtained from different graph models. Vital to this is a graph structure that is contingent on the data points. LPP discovers a projection that compliments this graph structure. In our theoretical analysis, we show how PCA, LDA, and LPP arise from the same principle applied to different choices of this graph structure.

It is worthwhile to focus some aspects of the proposed approach here:

1. While the Eigen faces technique purposes to preserve the global structure of the image space, and the Fisher faces technique goals to preserve the discriminating information; our Laplacian faces method aims to preserve the local structure of the image space. In various real-world classification problems, the local manifold structure is more important than the global Euclidean structure, especially when nearest-neighbour like classifiers are used for classification. LPP seems to have discriminating power even though it is unproven.

2. An effective subspace learning algorithm for face recognition should be able to find the nonlinear manifold structure of the face space. Our proposed Laplacian faces technique explicitly reflects the manifold structure which is modelled by an adjacency graph. Furthermore, the Laplacian faces are gained by finding the optimal linear approximations to the Eigen functions of the Laplace Beltrami operator on the face manifold. They imitate the intrinsic face manifold structures.

3. LPP shares several similar properties to LLE [18], such as a locality preserving character, though, their objective functions are completely unlike. LPP is achieved by finding the optimal linear approximations to the Eigen functions of the Laplace Beltrami operator on the manifold. LPP is linear, whereas LLE is nonlinear. Furthermore, LPP is defined everywhere, while LLE is defined only on the training data points and it is unclear how to evaluate the maps for new test points. In contrast, LPP may be merely applied to any new data point to discover it in the reduced representative space test points.

2 PCA and LDA

One approach to deal with the difficulty of extreme dimensionality of the image space is to reduce the dimensionality by combining features. Linear permutations are particular, attractive because they are simple to calculate and logically tractable. In effect, linear methods project the high-dimensional data onto a lower dimensional subspace.

Considering the problem of representing all of the vectors in a set of n-dimensional samples x_1, x_2,... x_n, with zero mean, by a single vector $y = \{y_1, y_2 \ldots . y_n\}$ such that y_i represents x_i. Precisely, we find a linear mapping from the d-dimensional space to a line. Without loss of generality, we represent the transformation vector by w. That is, $w^T x_i = y_i$. In reality, the magnitude of w is of no real significance because it just scales y_i. In face recognition, each vector x_i denotes a face image.

Different objective functions will yield different algorithms with different properties. PCA aims to extract a subspace in which the variance is maximized. Its objective function is as follows:

$$\max_w \sum_{i=1}^{n} (y_i - \bar{y})^2 , \qquad (1)$$

$$\bar{y} = 1/n \sum_{i=1}^{n} y_i . \qquad (2)$$

The output set of principal vectors $w_1, w_2, \ldots . w_k$ is an orthonormal set of vectors representing the Eigenvectors of the sample covariance matrix associated with the $k < d$ largest Eigenvalues.

3 Learning a locality preserving subspace

PCA and LDA aim to preserve the global structure. Though, in many real-world applications, the local structure is more important. In this section, we describe Locality Preserving Projection (LPP) [9], a new algorithm for learning a locality preserving subspace. The comprehensive derivation and theoretical explanations of LPP can be traced [9]. LPP looks for preserving the basic geometry of the data and local structure. The objective function of LPP is as follows:

$$\min \sum_{ij} (y_i - y_j)^2 w_{ij} ,$$

where y_i is the one-dimensional representation of x_i and the matrix S is a similarity matrix. A possible way of defining S is as follows:

$$w_{ij} = \left\{ \exp(-\|x_i - x_j\|^2 \; / \; t \right.$$

0 otherwise }

The objective function with our choice of symmetric weights $w_{ij}(w_{ij} = w_{ji})$ incurs a heavy penalty if neighbouring points x_i and x_j are mapped far apart, i.e., if$(y_i - y_j)^2$ is large. Therefore, minimizing it is an attempt to ensure that, if x_i and x_j are "close," then y_i and y_j are close as well. Following some simple algebraic steps, we see that :

$$\tfrac{1}{2}\sum_{ij} (y_i - y_j)^2 \, W_{ij}$$

$$= \tfrac{1}{2}\sum_{ij} (w^T x_i - w^T x_j)^2 \, W_{ij}$$

$$= w^T XDX^T w - w^T X W X^T w$$

$$= w^T X (D - W) X^T w$$

$$= w^T XLXTw$$

where $X = [x_1, x_2 ..... x_n]$, and D is a diagonal matrix; its
entries are column (or row since S is symmetric) sums of W, $D_{ii} = \sum_j W_{ji}$. L= D - W is the Laplacian matrix [6].

4 Locality Preserving Projections

4.1. The linear dimensionality reduction problem

The basic problem of linear dimensionality reduction is the follows: Given a set $x_1, x_2, \ldots x_m$ in R^n, find a transformation matrix A that maps these m points to a set of points $y_1, y_2, \ldots y_m$ in R^l ($l << n$), such that y_i "represents" x_i, where $y_i = A^T x_i$.
Our method is of particular applicable in the special case where $x_1, x_2 \ldots x_m \in M$ and M is a nonlinear manifold embedded in R^n.

4.2. The algorithm

Locality Preserving Projection (LPP) is a linear approximation of the nonlinear Laplacian Eigen map [2]. The algorithmic procedure is formally stated below:

1. Constructing the adjacency graph: Let G denote a graph with m nodes. We put an edge between nodes i and j if x_i and x_j are "close". There are two variations:

(a) ε-neighbourhoods: [parameter $\varepsilon \in R$] Nodes i and j are connected by an edge if $\| x_i - x_j \|^2 < \varepsilon$ where the norm is the usual Euclidean norm in $\mathbf{R^n}$.

(b) k nearest neighbours: [parameter $k \in N$] Nodes i and j are connected by any edge if i is among k nearest neighbours of j or j is among k nearest neighbours of i.

Note: The process of constructing an adjacency graph outlined above is correct if the data actually lie on a low dimensional manifold. In general, though, one might take a more practical viewpoint and construct an adjacency graph based on any principle (for example, perceptual similarity for natural signals, hyperlink structures for web documents, etc.). Once such an adjacency graph is obtained, LPP will try to optimally preserve it in choosing projections.

2. Choosing the weights: Here, as well, we have two variations for weighting the edges. W is a sparse symmetric m×m matrix with W_{ij} having the weight of the edge joining vertices i and j, and 0 if there is no such edge.

(a) Heat kernel: [parameter $t \in R$]. If nodes i and j are connected, put

$$W_{ij} = e^{(-\| x_i - x_j \|^2 / t)}$$

The justification for this choice of weights can be traced back to [2].

(b) Simple-minded: [No parameter]. $W_{ij} = 1$ if and only if vertices i and j are connected by an edge.

3. Eigen maps: Compute the Eigenvectors and Eigenvalues for the generalized Eigenvector problem:

$$XLX^T \mathbf{a} = \lambda XDX^T \mathbf{a}$$

where D is a diagonal matrix whose entries are column (or row, since W is symmetric) sums of W, $D_{ii} = \sum_j W_{ji}$. $L = D - W$ is the Laplacian matrix. The i^{th} column of matrix X is x_i.

Let the column vectors $a_0 \ldots\ldots a_{l-1}$ be the solutions of equation (1), ordered according to their Eigenvalues, $\lambda_0 < \ldots\ldots < \lambda_{l-1}$. Thus, the embedding is as follows:

$x_i \rightarrow y_i = A^T x_i$, $A = (a_0, a_1, \ldots\ldots a_{l-1})$

where y_i is a l-dimensional vector, and A is a $n \times l$ matrix.

5 Geometrical Justification

The Laplacian matrix L= (D - W) for finite graph, or [4], is analogous to the Laplace Beltrami operator $\mathcal{L}$ on compact Riemannian manifolds. While the Laplace Beltrami operator for a manifold is generated by the Riemannian metric, for a graph it comes from the adjacency relation.

Let $\boldsymbol{M}$ be a smooth, compact, **d**-dimensional Riemannian manifold. If the manifold is embedded in R^n, the Riemannian structure on the manifold is induced by the standard Riemannian structure on R^n. We are looking here for a map from the manifold to the real line such that points close together on the manifold get mapped close together on the line. Let $\boldsymbol{f}$ be such a map. Assume that $\boldsymbol{f} : M \rightarrow R$ is twice differentiable.

Belkin and Niyogi [2] showed that the optimal map preserving locality can be found by solving the following optimization problem on the manifold:

$$\underset{\|f\|_{L^2(M)}=1}{\arg\min} \int_M \|\nabla f\|^2$$

which is equivalent to

$$\underset{\|f\|_{L^2(M)}=1}{\arg\min} \int_M \mathcal{L}(f)\, f$$

where the integral is taken with respect to the standard measure on a Riemannian manifold. $\mathcal{L}$ is the Laplace Beltrami operator on the manifold, i.e. $\mathcal{L} f = -\operatorname{div}\nabla(f)$. Thus, the optimal f has to be an Eigen function of $\mathcal{L}$. The integral $\int_M \mathcal{L}(f) f$ can be discretely approximated by $[f(X), Lf(X)] = f^T(X)\, Lf(X)$ on a graph, where

$$f(X) = [f(x_1), f(x_2), \ldots\ldots f(x_m))]^T, \quad f^T(X) = [f(x_1), f(x_2), \ldots.. f(x_m))]$$

If we restrict the map to be linear, i.e. $f(x) = a^T x$, then we have

$f(X) = X^T a \rightarrow [f(X), Lf(X)] = f^T(X) L f(X) = a^T X L X^T a$

The constraint can be computed as follows,

$$\|f\|_{L^2(M)} = \int_M (a^T x)^2\, dx = \int_M (a^T x\, x^T a)\, dx = a^T \left(\int_M x x^T dx\right) a$$

where dx is the standard measure on a Riemannian manifold. By spectral graph theory [4], the measure dx directly corresponds to the measure for the graph which is the degree of the vertex, i.e. D_{ii}. Thus, $\|f\|_{L^2(M)}$ can be discretely approximated as follows,

$$\|f\|^2_{L^2(M)} = a^T \left(\int_M x x^T dx\right) a \approx a^T \left(\sum_i x x^T D_{ii}\right) a = a^T X D X^T a$$

Finally, we conclude that the optimal linear projective map, i.e. $f(x) = a^T x$, can be obtained by solving the following objective function,

$$\arg\min a^T X L X^T a$$
$$a^T X D X^T a = 1$$

These projective maps are the optimal linear approximations to the Eigen functions of the Laplace Beltrami operator on the manifold. Therefore, they are capable of discovering the nonlinear manifold structure.

6 Experimental Results

Some simple mock examples given in [9] show that LPP can have additional discriminating power than PCA and be less subtle to outliers. In this section, numerous experiments are carried out to demonstrate the effectiveness of our proposed Laplacian faces technique for face representation and recognition.

6.1 Face Representation Using Laplacian faces

As we defined earlier, a face image can be represented as a point in image space. A typical image of size m × n describes a point in m × n - dimensional image space. On the other hand, due to the undesirable variations resulting from changes in lighting, facial expression, and pose, the image space might not be an ideal space for visual representation. The images of faces in the training set are used to learn such a locality preserving subspace. The subspace is covered by a set of Eigenvectors of (35), i.e., $w_0, w_1, \ldots\ldots, w_{k-1}$. We can show the Eigenvectors as images. These images may be called Laplacian faces. Using the face database as the training set, we present the first 10 Laplacian faces in figure, in conjunction with Eigen faces and Fisher faces. A face image can be mapped into the locality preserving subspace by using the Laplacian faces. It is interesting to note that the Laplacian faces are in some way similar to Fisher faces.

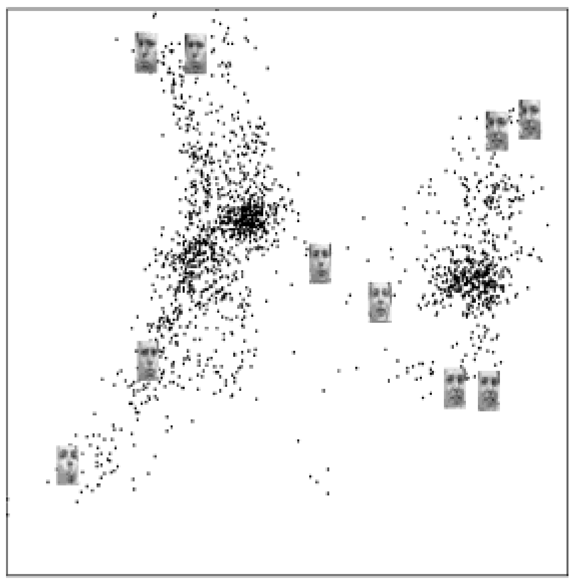

Figure 1: Distribution of the 10 testing samples in the reduced representation subspace. As can be seen, these testing samples optimally find their coordinates which reflect their intrinsic properties, i.e., pose and expression.

When the Laplacian faces are created, face recognition [2],[14], [28], [29] becomes a pattern classification task. In this section, we examine the performance of our proposed Laplacian faces method for face recognition. The system performance is equated with the Eigen faces method [28] and the Fisher faces method [2], two of the most popular linear methods in face recognition. In this study, face database was tested. That one is the PIE (pose, illumination, and expression) database. In all the experiments, pre-processing to trace the faces was applied. Original images were normalized (in scale and orientation) such that the two eyes were aligned at the same position. Then, the facial areas were cropped into the final images for matching. Figure 2 shows the original image and the cropped image. The size of each cropped image in all the experiments is 32 × 32 pixels, with 256 grey levels per pixel. Thus, each image is represented by a 1,024-dimensional vector in image space.

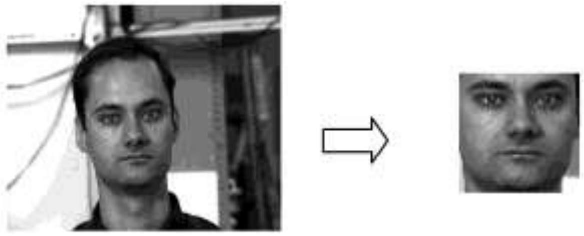

Figure 2: The original face image and the cropped image

The facts of our methods for face detection and alignment can be found in [30], [32]. No further pre-processing is done. Different pattern classifiers have been applied for face recognition, including nearest-neighbour [2], Bayesian [15], Support Vector Machine[17], etc. In this paper, we apply the nearest-neighbour classifier for its simplicity. The Euclidean metric is used as our distance measure.

In short, the recognition process has three steps. First, we calculate the Laplacian faces from the training set of face images and then the new face image to be recognized is projected into the face subspace spanned by the Laplacian faces. Finally, the new face image is identified by a nearest-neighbour classifier.

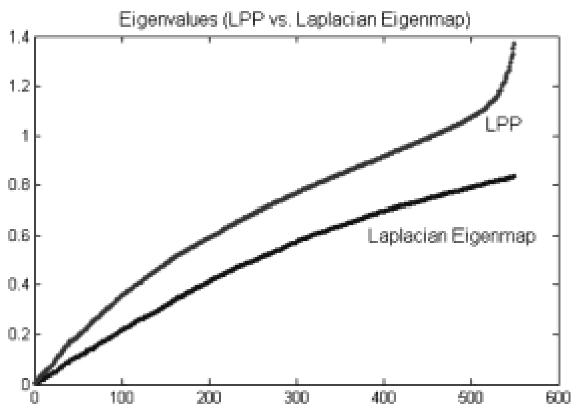

Figure 3: The Eigenvalues of LPP and Laplacian Eigen map.

6.1.1 PIE Database

The PIE face database contains 68 subjects with 41,368 face images as a whole. The face images were taken by 13 synchronized cameras and 21 flashes, under varying pose, illumination, and expression. We used 170 face images for each individual in our experiment, 85 for training and the other 85 for testing. Figure 4 shows some of the faces with pose, illumination and expression variations in the PIE database.

Figure 4: The sample cropped face images of one individual from PIE database. The original face images are taken under varying pose, illumination, and expression

Table 1 shows the recognition results. As can be seen, Fisher faces perform comparably to our algorithm on this database, while Eigen faces performs poorly. The error rate for Laplacian faces, Fisher faces, and Eigen faces are 4.6 per cent, 5.7 per cent, and 20.6 per cent, respectively. Figure 5 shows a plot of error rate versus dimensionality reduction. As can be seen, the error rate of our Laplacian faces method decreases fast as the dimensionality of the face subspace increases, and achieves the best result with 110 dimensions. There is no significant progress if more dimensions are used. Eigen faces achieves the best result with 150 dimensions. For Fisher faces, the dimension of the face subspace is bounded by c - 1, and it achieves the best result with c -1 dimensions. The dashed horizontal line in the figure shows the best result obtained by Fisher faces.

Approach	Dims	Error Rate
Eigenfaces	150	20.6%
Fisherfaces	67	5.7%
Laplacianfaces	**110**	**4.6%**

TABLE 1: Performance Comparison on the PIE Database

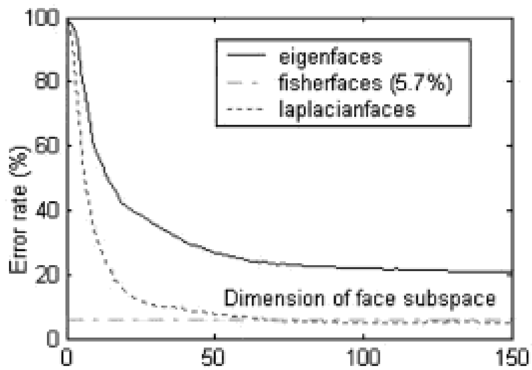

Figure 5: Recognition accuracy versus dimensionality reduction on PIE database

6.2 Discussion

These experiments on the database have been system-atically performed. These experiments disclose a number of remarkable points :

1. All these three approaches performed well in the optimal face subspace than in the original image space.

2. In all the three experiments, Laplacian faces consis-tently performs better than Eigen faces and Fisher faces.
These experiments also demonstrate that our algorithm is especially suitable for frontal face images. Likewise, our algorithm takes advantage of more training samples, which is important to the real-world face recognition systems.

3. Equating to the Eigen faces method, the Laplacian faces method scrambles more discriminating information in the low-dimensional face subspace by preserving indigenous structure which is more important than the global structure for classification, especially when nearest-neighbour-like classifiers are used. In effect, if there is a reason to believe that Euclidean distances ($\|x_i - x_j\|$) are significant only if they are small (local), then our algorithm finds a projection that respects such a belief.

7 Conclusion and future work

The manifold ways of face analysis (representation and
recognition) are introduced in this paper in order to identify the basic nonlinear manifold structure in the way of linear subspace learning. To the best of our knowledge, this is the first devoted work on face representation and recognition which unambiguously reflects the manifold structure. The manifold structure is estimated by the adjacency graph computed from the data points. Using the notion of the Laplacian of the graph, we then compute a transformation matrix which maps the face images into a face subspace. We

call this the Laplacian faces approach. The Laplacian faces are obtained by finding the optimal linear approximations to the Eigen functions of the Laplace Beltrami operator on the face manifold. This linear conversion optimally preserves local manifold structure.

One of the vital problems in face manifold learning is to approximate the inherent dimensionality of the nonlinear face manifold, or, degrees of freedom. We know that the dimensionality of the manifold is equal to the dimensionality of the local tangent space. Some previous works [35], [36] show that the local tangent space can be estimated using points in a neighbour set. Hence, one possibility is to approximate the dimensionality of the tangent space.

An additional possible extension of our work is to study the use of the unlabelled samples. It is important to note that the work presented here is a general method for face analysis (face representation and recognition) by discovering the underlying face manifold structure. Learning the face manifold (or learning Laplacian faces) is principally an unverified learning process. Meanwhile the face images are supposed to exist in a sub manifold embedded in (Placeholder1) a high-dimensional ambient space, we believe that the unlabelled samples are of great value.

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